# Rossmoyne Senior High School

### Year 12 Trial WACE Examination, 2014

### Question/Answer Booklet

**SOLUTIONS**

# MATHEMATICS: SPECIALIST 3C/3D

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student Number: In figures |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

## Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet (retained from Section One)

##### *To be provided by the candidate*

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

## Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
| Section One:  Calculator-free | 7 | 7 | 50 | 50 | 33⅓ |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 100 | 66⅔ |
|  | | | **Total** | 150 | 100 |

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

* Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
* Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

1. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
2. It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed (100 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (5 marks)

If  and , then  can be written in the form .

(a) Determine the values of  and . (3 marks)

Using CAS, 

Hence



(b) Determine the equation of the tangent to  at the point (2, -16). (2 marks)



Question 9 (5 marks)

A triangle has vertices at (5, -4, -7), (7, 2, -9) and (-3, 0, 5).

(a) Determine the exact length of side . (1 mark)



(b) Determine the size of , correct to the nearest degree. (2 marks)



(c) The point  lies on the side such that the length is three times the length. Determine the vector , where  is the origin. (2 marks)



Question 10 (8 marks)

(a) Sketch the following polar graphs on the axes below for .

(i) . (1 mark)

(ii) . (1 mark)

(iii) . (2 marks)



(b) Given that  and , point  is the intersection of  and , and point  is the intersection of  and .

Determine the exact polar coordinates of  and  in the form . (2 marks)



(c) Determine the distance , rounded to three significant figures. (2 marks)



Question 11 (6 marks)

A triangle has vertices (1, 1), (3, 1) and (3, 4).

(a) Triangle is transformed to (1, -1), (3, -1) and (3, -4). Describe this transformation geometrically and state the  matrix that will transform  to.

(2 marks)

Reflection in the line .



(b) Triangle is dilated by a scale factor of ten about the origin and then rotated 30° clockwise about the origin to triangle.

(i) Determine the single  matrix that will transform  to . (2 marks)



(ii) Determine the area of triangle . (2 marks)

Original area of  is 3 sq units.

Reflection and rotation will not change area, but dilation will increase area by a factor of .

Area is 300 sq units.

Question 12 (8 marks)

The line  and the curve  are shown below. An area in the first quadrant bounded by the line, the curve and the -axis has been shaded.



(a) The tangents to the curve at  and  intersect at the point . Determine the values of  and . (4 marks)



(b) If the shaded region in the first quadrant, as shown on the diagram above, has an area of 10 square units, determine the value of , giving your answer to two decimal places.

(4 marks)



(Ignore  as these would not

give area shown in first quadrant).

Question 13 (8 marks)

In a herd of 3 500 cattle, 15 animals are known to have a disease. If left unchecked, the number of diseased cattle in the herd, , will increase at a rate given by



where  is the number of days since the initial 15 animals were discovered to have the disease.

(a) Use the above information to write  as a function of . (3 marks)

 (or using CAS dSolve function)

(b) How long will it take for more than 4% of the herd to have the disease? (2 marks)



Approximately 18 days.

(c) After 40 days, measures are taken to prevent any more cattle contracting the disease, and the use of medication will decrease the number of diseased cattle so that .

How long will it take from this time for less than 1% of the herd to have the disease?

(3 marks)



Approximately 14 days.

Question 14 (10 marks)

The motion of a small body moving in a straight line was recorded by a video camera for 40 seconds. An analysis of the motion showed that the distance,  cm, of the small body from a fixed point  on its path  seconds after recording began was given by .

(a) Show that the body is undergoing simple harmonic motion. (2 marks)



(b) Determine the initial displacement and velocity of the body. (2 marks)



(c) State the period and amplitude of the motion. (2 marks)



(d) Determine that the maximum speed of the body during its motion. (2 marks)



(e) Determine the total distance travelled by the body during the 40 seconds of filming.

(2 marks)

In 40 seconds, body will travel 5 complete cycles (period is 8 seconds).

Distance travelled in one cycle is 20 cm, so total distance travelled is 100 cm.

Question 15 (9 marks)

(a) Sketch in the complex plane the region satisfying the two inequalities given by

 and  (5 marks)



(b) If  is the complex number that satisfies both inequalities given in (a), determine the minimum and maximum vales of . (4 marks)

Minimum value when .

Maximum value when .

Question 16 (8 marks)

A plane has equation , where  and .

(a) If 

(i) show that  is perpendicular to both  and . (1 mark)



Hence both perpendicular as dot products are zero.

(ii) determine the equation of the plane in the form . (2 marks)



(b) Determine the coordinates of the point where the line  meets the plane.

(5 marks)



Question 17 (8 marks)

Consider the life cycle of a head louse that has a life span of 4 weeks. In a simplified model, the first week of life is spent as an egg. The egg hatches into a nymph during the second week and for the last two weeks of its life it is an adult, when it can lay several eggs per day. In a model to examine a lice infestation, the following information was assumed.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Age in weeks | 0-1 (Egg) | 1-2 (Nymph) | 2-3 (Adult) | 3-4 (Adult) |
| Eggs laid per week | 0 | 0 | 30 | 40 |
| Percentage surviving | 20% | 20% | 10% | 0 |

(a) Write down a Leslie matrix to model changes in the age class distribution from the above information. (2 marks)



An infestation of lice begins with just two adults, one aged 2-3 weeks and one aged 3-4 weeks in the head of a child.

(b) Determine the expected number of eggs, nymphs and adults in the child's head after

(i) 2 weeks. (2 marks)

 4 eggs, 14 nymphs and 0 adults.

(ii) 12 weeks. (2 marks)

 9 eggs, 11 nymphs and 5 adults.

(c) If the child commenced a treatment that removed half of the eggs present each week, describe the effect this has on the long term population of head lice. (2 marks)

With the initial model, the population of head lice increases unrealistically after a large number of weeks, but removing half of the eggs each week leads to the population ceasing to exist after about 30 weeks.

Question 18 (9 marks)

Let the  be the area of the region between two concentric circles of radii  and  () at any time  seconds.  is increasing at a constant rate of 4 cms-1 and when , 60 cm and 20 cm.

(a) If  is increasing at a constant rate of 5 cms-1, determine

(i) the rate of increase of  when . (3 marks)



(ii) the ratio of  to  when  begins to decrease. (2 marks)



(iii) the time at which  is zero. (1 mark)



(b) If the area is fixed, determine the rate of increase of  when 90 cm. (3 marks)



Question 19 (7 marks)

(a) If , determine  and the values of  when  and when . (2 marks)





(b) Explain why  for all . (2 marks)

From the graph of  in the first quadrant, it is clear that the area under the curve between 1 and  added to the area between  and  will be the same as the area between 1 and .

(c) The natural logarithm of  can be expressed as  for . Use this definition, together with the substitution , to prove that  for all .

Do **not** use any laws of logarithms. (3 marks)



Question 20 (9 marks)

(a) Prove by contradiction that the last digit of , where  is a positive integer, will never be zero. (3 marks)

Assume that the last digit of  is 0, so that , where  is an integer.

Since  is divisible by 5, then so must , but this is impossible, and so our original assumption is contradicted, meaning that the last digit of  can never be zero.

(b) The sequence of hexagonal numbers, , is given by the recursive rule , .

(i) Show that the third hexagonal number is 15. (1 mark)



(ii) Prove by induction that the  hexagonal number, , can also be found using the explicit rule , . (5 marks)

When , , and so the rule is true for this case.

If it is assumed that the rule is true for , i.e. , then it is necessary to prove the result for , i.e. .

Now, using the original recursive rule gives:



Thus shown to be true for  and as the truth for  implies the result for  it follows that the conjecture is true for all positive integers.

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

This examination paper may be freely copied, or communicated on an intranet, for non-commercial purposes within educational institutes that have purchased the paper from WA Examination Papers provided that WA Examination Papers is acknowledged as the copyright owner. Teachers within Rossmoyne Senior High School may change the paper provided that WA Examination Paper's moral rights are not infringed.

Copying or communication for any other purposes can only be done within the terms of the Copyright Act or with prior written permission of WA Examination papers.

*Published by WA Examination Papers*

*PO Box 445 Claremont WA 6910*